

# Klett Inversion of Simulated LIDAR Signals

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October 15, 2001

## Abstract

A Klett inversion of simulated LIDAR signals is shown for 4 model atmospheres. The atmospheres differ in having two (different) vertical distributions of the aerosols:  $(h_m, h_s) = (0\text{m}, 1200\text{m})$  and  $(400\text{m}, 800\text{m})$ , and two (different) values for the aerosol phase function at  $180^\circ$ :  $\frac{1}{\sigma^a} \left( \frac{d\sigma^a}{d\Omega} \right)_{180^\circ} = 0.05$  and  $0.025$ . For all of the model atmospheres the aerosol horizontal extinction length,  $\Lambda^a(0) = 20\text{km}$  (at the elevation of the fluorescence detectors). The simulation allows the Klett *k-parameter* to be evaluated *versus* height. Results are given assuming Klett *k-parameter* values of 0.7, 0.8 and 0.9.

## 1 Introduction

In an previous note [1] we studied the question: are LIDAR *inversion* techniques able to achieve an aerosol transmission uncertainty correction meeting the Auger goal of  $\sim 10\%$  uncertainty? As the earlier note used only one of the common LIDAR inversion techniques, that due to Fernald [2], it is of interest to know how the technique due to Klett [3] performs on the same (ideal) simulated LIDAR data [1].

The concept of this study is to simulate an ideal LIDAR and an (ideal) 1-dimensional atmosphere: where the molecular and aerosol components are both known and easily varied. The LIDAR signals from this simulation are then analyzed following Klett [3].

## 2 Model Parameters and Analysis Results

The atmosphere is modeled using the US Standard Atmosphere [4] plus Rayleigh scattering for the molecular component. The aerosols are modeled using 3 parameters:

- aerosol horizontal extinction length,  $\Lambda^a(0)$  at the height of the fluorescence detectors,  $z \equiv 0$  (nominally 1500m above sea level),
- aerosol mixing height,  $h_m$ ,
- aerosol scale height,  $h_s$ .

Thus the aerosol extinction length *versus* height,  $\Lambda^a(z)$ , is given by:

$$\Lambda^a(z) = \Lambda^a(0) \quad (z \leq h_m)$$

and:

$$\Lambda^a(z) = \Lambda^a(0) \cdot e^{(z-h_m)/h_s} \quad (z > h_m)$$

In this notation the aerosol optical depth,  $\tau^a(z)$ , is:

$$\tau^a(z) = \int_0^z \frac{dz}{\Lambda^a(z)}$$

which for  $z \gg h_m + h_s$  gives  $\tau^a = (h_m + h_s)/\Lambda^a(0)$ . Values used in the study are:  $\Lambda^a(0) = 20\text{km}$  and  $h_m = 0\text{m}$  and  $h_s = 1200\text{m}$  or  $h_m = 400\text{m}$ ,  $h_s = 800\text{m}$ . Vales used for the aerosol phase function at  $180^\circ$  are 0.025 or 0.05. The (resulting) model aerosol extinction lengths,  $\Lambda(z)$ , *versus* height are shown in Fig. 1.

The Klett analysis requires the LIDAR signal *versus* time, the density of the molecular atmosphere *versus* height, the Rayleigh cross section and a power law parameter,  $k$  [3], where:

$$\beta(z) \propto \sigma(z)^k$$

with:

$$\beta(z) = \frac{1}{\Lambda^m(z)} \left[ \frac{1}{\sigma^m} \left( \frac{d\sigma^m}{d\Omega} \right) \right]_{180^\circ} + \frac{1}{\Lambda^a(z)} \left[ \frac{1}{\sigma^a} \left( \frac{d\sigma^a}{d\Omega} \right) \right]_{180^\circ}$$

and:

$$\sigma(z) = \frac{1}{\Lambda^m(z)} + \frac{1}{\Lambda^a(z)}$$

Superscripts “a” and “m” correspond to the aerosol (Mie) and molecular (Rayleigh) terms respectively.

The LIDAR is modeled with a 5mJ/pulse, 355nm laser, 0.5m<sup>2</sup> mirror, overall efficiency (photons to P.E.s) of 15% and 10Mhz sampling. These results are all from simulated operation at an angle  $\alpha = 30^\circ$  (to the horizontal).

As noted above, the Klett analysis (of the simulated LIDAR data) needs a value for  $k$ . Klett [3] notes that values in the range of 0.67 to 1 are found in the literature. Values used in other studies include 0.7 [5] and 1.0 [6]. For the simulated atmosphere,  $k$  can be evaluated using  $d\beta(z)/dz = k \cdot d\sigma(z)/dz$ . This is plotted in Fig. 2 for the four model atmospheres. Clearly  $k$  is not a constant as values of  $k$  vary between 0.4 to 1.0. Somewhat arbitrary values of  $k = 0.7, 0.8$  and  $0.9$  were used in this study.

The Klett analysis steps inward from the starting point. This study starts at an elevation of  $\sim 7000$ m and steps inwards from there. The output of the analysis is the aerosol reconstructed extinction length *versus* height,  $\Lambda^r(z)$ . The values of  $z$  (in this toy study) correspond to each digitization of the simulated LIDAR signal; for  $\alpha = 30^\circ$  and 100ns sampling the  $z$ -binning is 7.5m.

The results for the four model atmospheres, each analyzed with three values of  $k$ , are shown in Fig. 2 - 5. In each case part “a” of the figure shows the reconstructed aerosol extinction length,  $\Lambda^r(z)$ , *versus* height. These can be compared with the input extinction length in Fig. 1. Part “b” of the figure shows the fractional aerosol transmission correction error,  $dT/T$ , (from our combined simulation plus analysis) for the three assumed values of  $k$ .

As with the Fernald analysis [1], different values of the input parameter, in this case  $k$ , result in different values of the aerosol extinction length at any given height (*e.g.* 500m). Unlike the Fernald analysis the shapes of the aerosol extinction length,  $\Lambda^r(z)$  (see Fig. 3a, 4a, 5a and 6a) differ significantly from the input (see Fig. 1). Furthermore it is not clear what value of  $k$  is most appropriate. If we choose the reconstruction that extrapolates most closely with the (input) aerosol phase function at ground level,  $z = 0$ , this corresponds to  $k = 0.7$  in all cases. However  $k = 0.7$  results in the largest values for  $dT/T$  in Fig. 3b and 4b. Furthermore, to match the (input) aerosol phase function at ground level,  $z = 0$ , values of  $k < 0.7$  are needed for the atmospheres with aerosol phase function at  $180^\circ = 0.025$  (as also suggested by Fig. 2). In this case if  $k = 0.6$  is used there is a better match between  $\Lambda^r(z)$  and the (input)

aerosol phase function at ground level but much larger values for  $dT/T$  are found (not shown).

### 3 Summary

Backscattered LIDAR data are simulated in a atmosphere including molecular and aerosol components. The aerosol extinction length *versus* height above the LIDAR is extracted following Klett. The simulation allows the Klett *k-parameter* to be evaluated *versus* height. Results are given assuming Klett *k-parameter* values of 0.7, 0.8 and 0.9. Uncertainties in choice of the *k-parameter* and the poor reproduction of the shape of the (input) aerosol extinction length bring into question the use of the Klett inversion technique to evaluate the aerosol transmission corrections for air fluorescence experiments.

### Acknowledgements

I want to thank Paul Sommers for urging me to do this study.

### References

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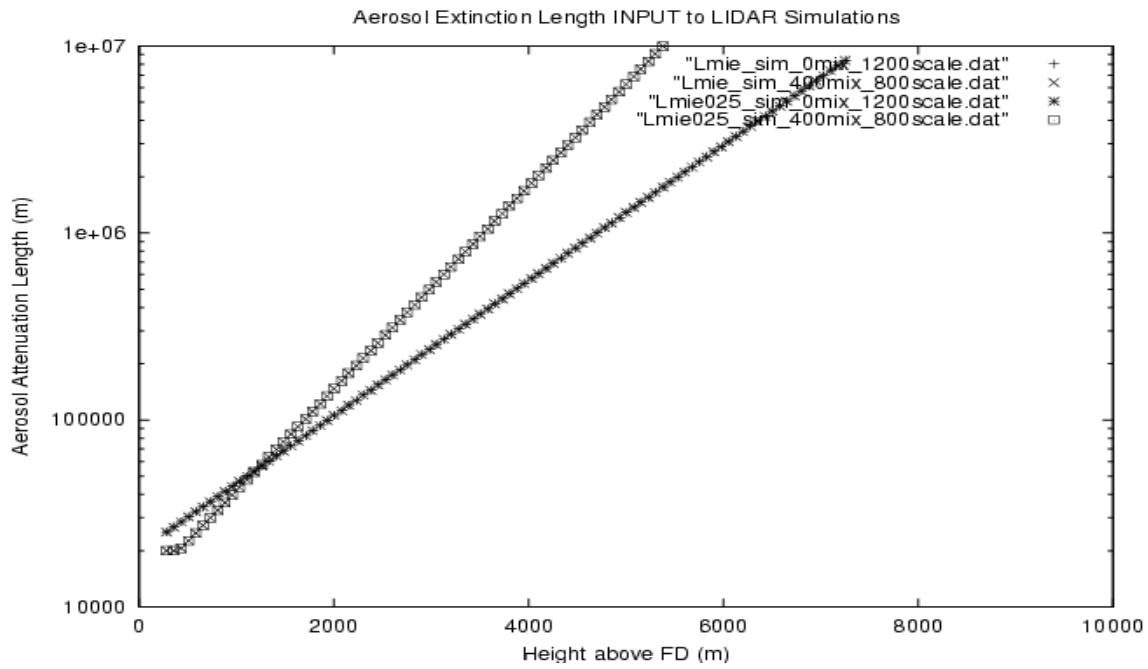


Fig. 1: Plot of aerosol extinction length,  $\Lambda^a(z)$ , versus height above the LIDAR (fluorescence detectors) for the four model atmospheres.

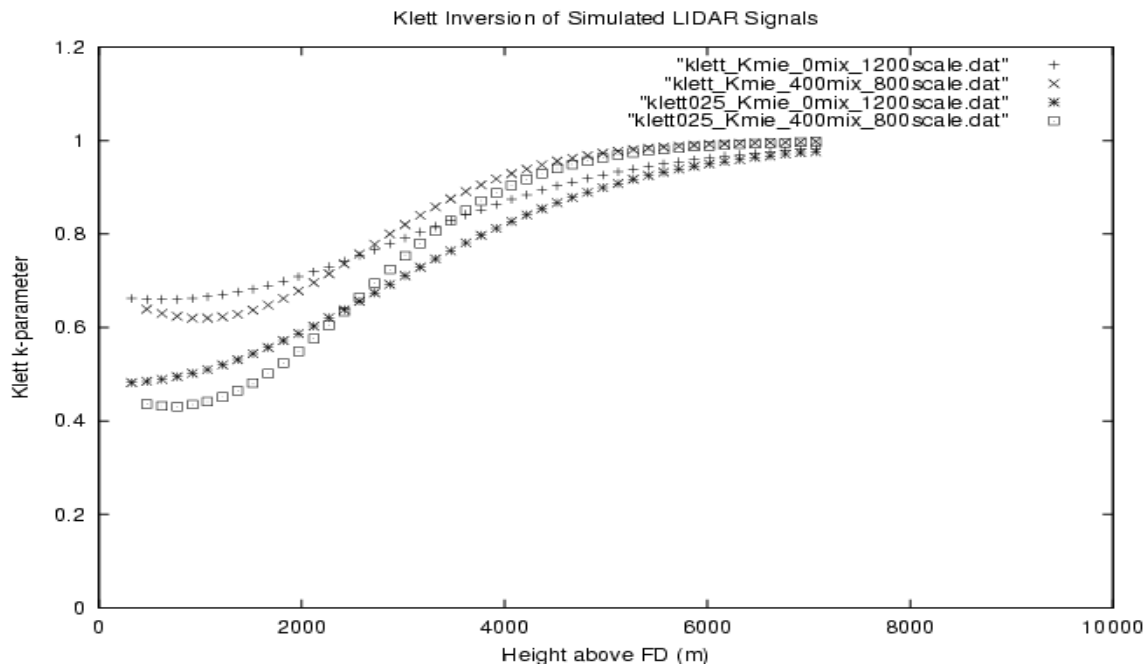


Fig. 2: Plot of Klett  $k$ -parameter versus height for the four model atmospheres.

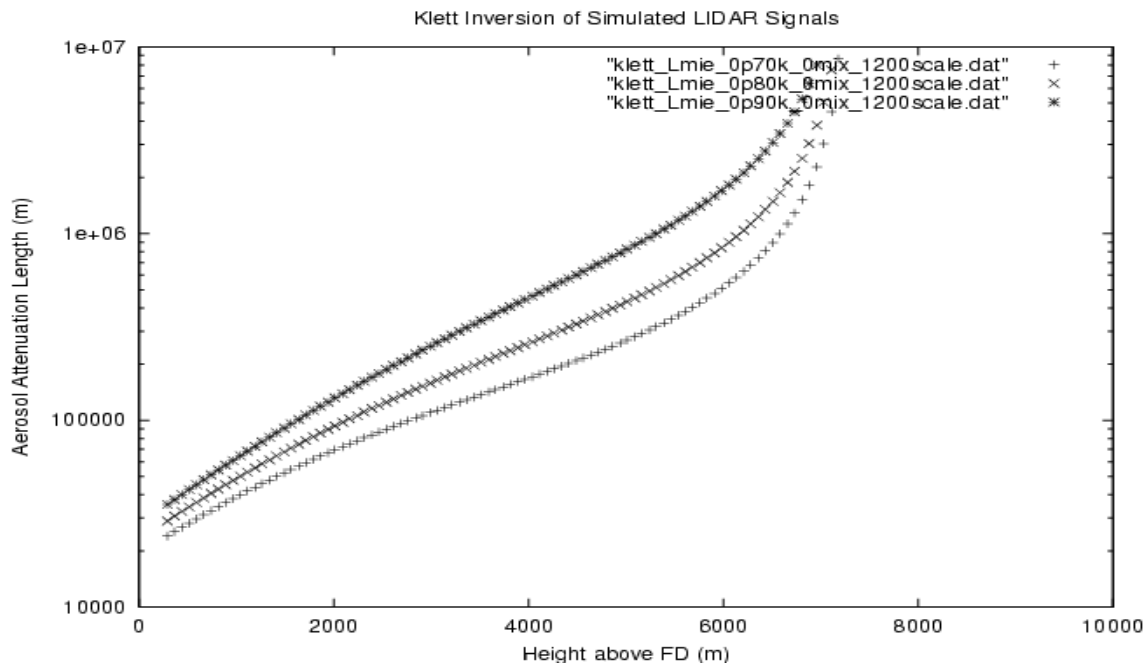


Fig. 3a: Reconstructed aerosol extinction length,  $\Lambda^r(z)$ , versus height for model atmosphere with  $h_m = 0\text{m}$ ,  $h_s = 1200\text{m}$ , and aerosol phase function at  $180^\circ = 0.05$ .

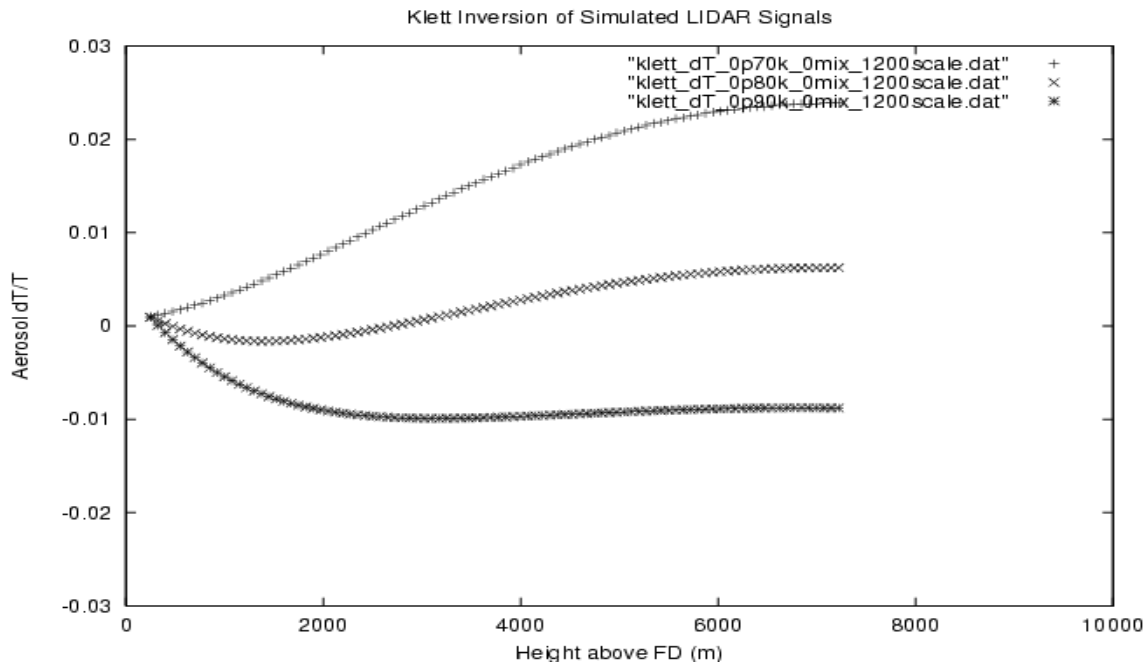


Fig. 3b: Plot of fractional error:  $dT/T = (1 - T^r/T^a)$ , where  $T^r$  is the reconstructed aerosol transmission factor and  $T^a$  is the actual (input) value, versus height,  $z$ .

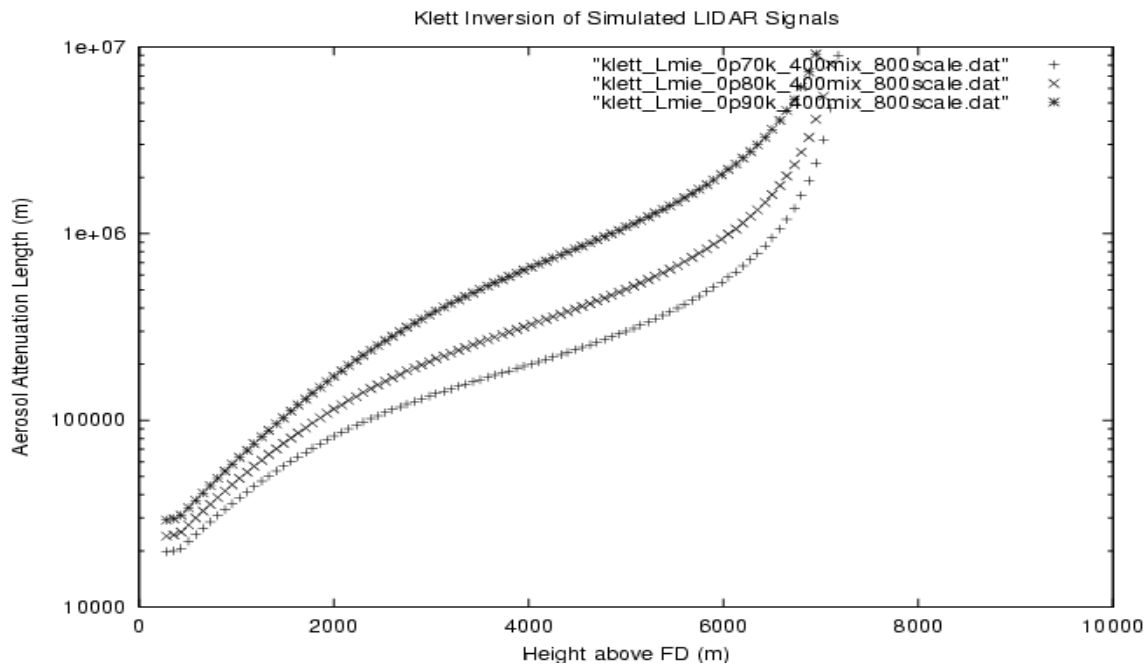


Fig. 4a: Reconstructed aerosol extinction length,  $\Lambda^r(z)$ , versus height for model atmosphere with  $h_m = 400\text{m}$ ,  $h_s = 800\text{m}$ , and aerosol phase function at  $180^\circ = 0.05$ .

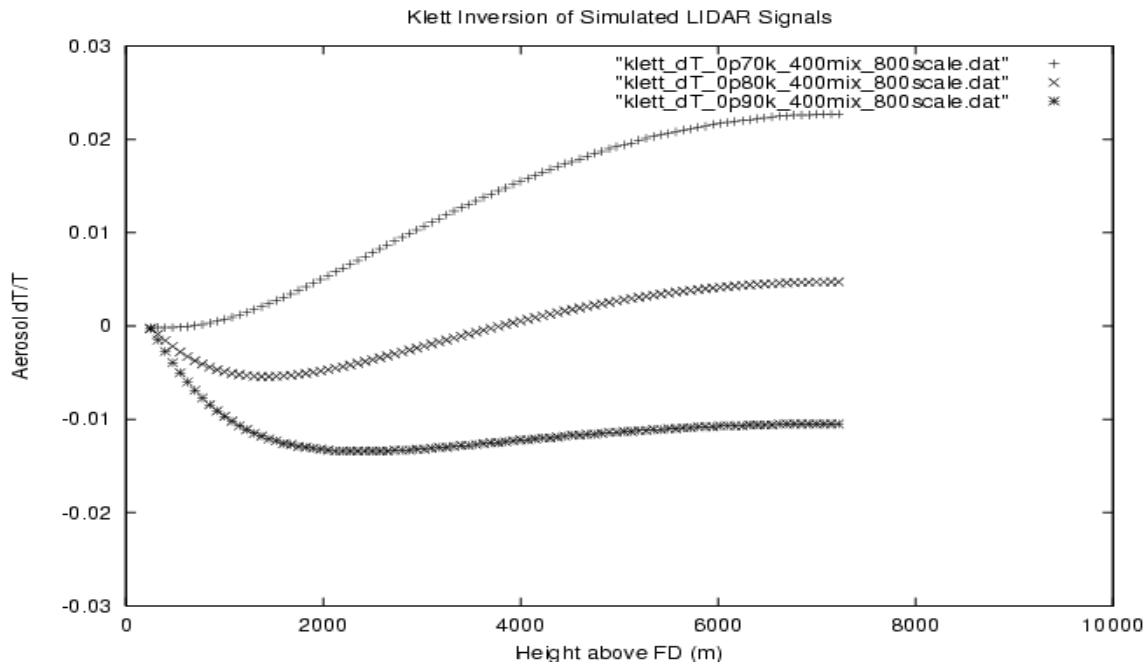


Fig. 4b: Plot of fractional error:  $dT/T = (1 - T^r/T^a)$ , where  $T^r$  is the reconstructed aerosol transmission factor and  $T^a$  is the actual (input) value, versus height,  $z$ .

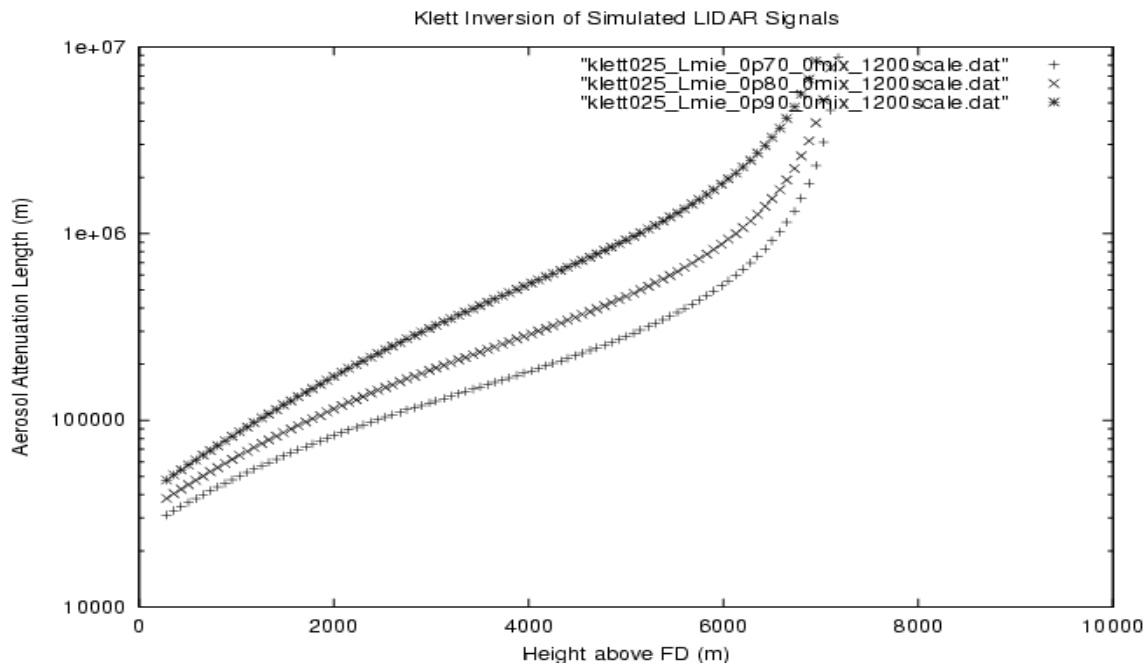


Fig. 5a: Reconstructed aerosol extinction length,  $\Lambda^r(z)$ , versus height for model atmosphere with  $h_m = 0\text{m}$ ,  $h_s = 1200\text{m}$ , and aerosol phase function at  $180^\circ = 0.025$ .

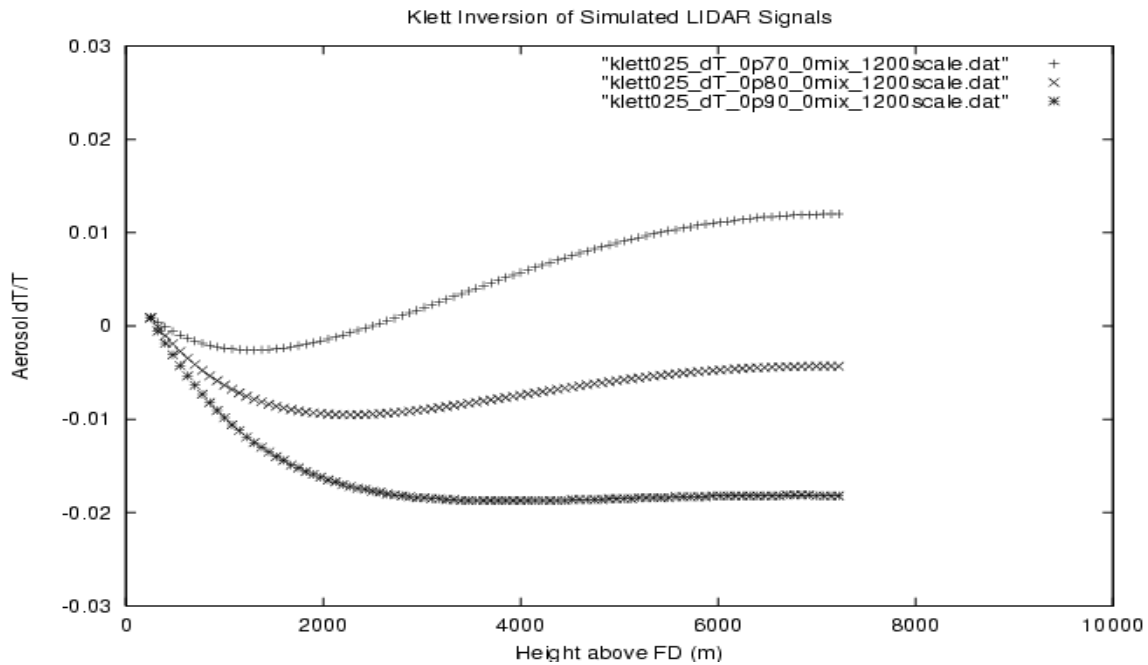


Fig. 5b: Plot of fractional error:  $dT/T = (1 - T^r/T^a)$ , where  $T^r$  is the reconstructed aerosol transmission factor and  $T^a$  is the actual (input) value, versus height,  $z$ .



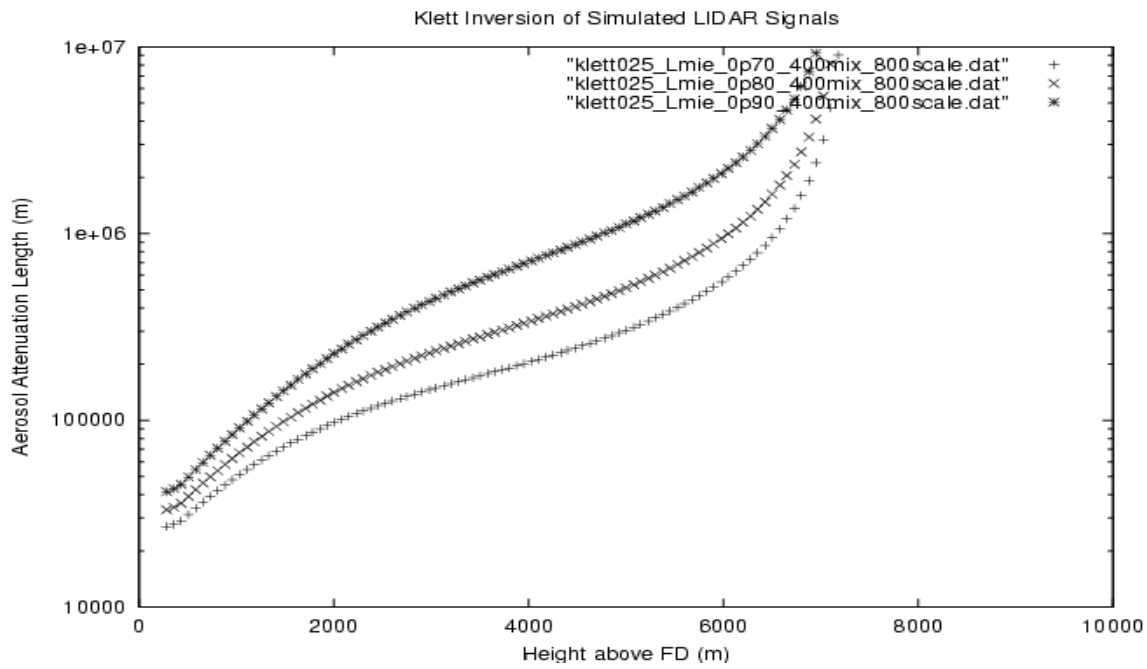


Fig. 6a: Reconstructed aerosol extinction length,  $\Lambda^r(z)$ , versus height for model atmosphere with  $h_m = 400\text{m}$ ,  $h_s = 800\text{m}$ , and aerosol phase function at  $180^\circ = 0.025$ .

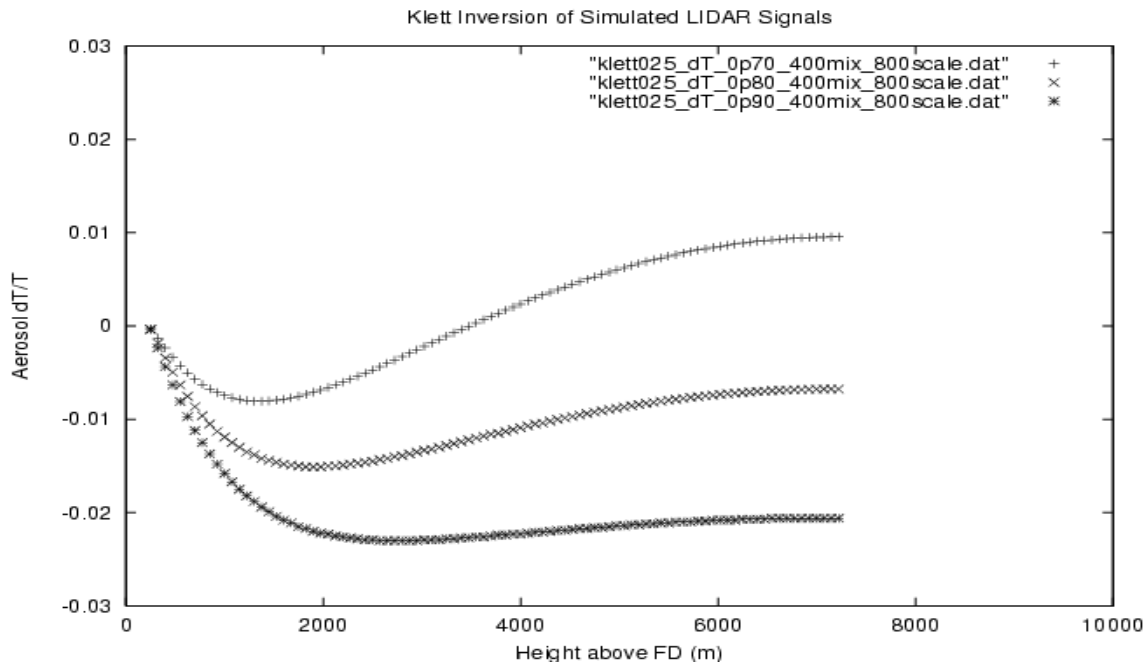


Fig. 6b: Plot of fractional error:  $dT/T = (1 - T^r/T^a)$ , where  $T^r$  is the reconstructed aerosol transmission factor and  $T^a$  is the actual (input) value, versus height,  $z$ .