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The Standard Atmosphere, $r_{Moliere}$ and Ground Array Systematic Errors

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Abstract

The particle density, $\rho(r)$, several hundred meters from the core of an extensive air shower provides a measure of the shower energy that is relatively insensitive to the depth of the initial interaction and to the cosmic ray composition. However the electro-magnetic contribution to $\rho(r)$ is sensitive to changes in the atmosphere. This study uses simple analytic shower theory to estimate variations in the electro-magnetic component of $\rho(r)$ resulting from temperature changes in the atmosphere over the course of the year. While this is only a toy study, temperature and pressure dependent variations in $\rho(r)$ are a potential source of *systematic* errors in the ground array measurements that Auger must be prepared to correct in any final analysis.

1. Introduction

The Auger measurement of extensive air showers will require careful monitoring of the atmosphere. The focus of this study is to provide an example illustrating how the variations in the air temperature over the course of a year can effect the ground array energy measurement.

To first order air temperature variations over the year change the local air density; the local pressure is constant. In Auger the ground array measurement of the shower energy will rely on the measurement of the particle density, $\rho(r)$, several hundred meters from the shower core. To see how air temperature variations impact the measurement of $\rho(r)$ it is instructive to use semi-analytic shower theory [1-4]. For an electro-magnetic shower the transverse shower profile is given by [3,4]:

$$\rho(r) = \frac{N_e(T_{eff})}{r_m^2} \cdot (\frac{r}{r_m})^{s-2} \cdot (1+\frac{r}{r_m})^{s-4.5} \cdot \frac{\Gamma(4.5-s)}{2\pi\Gamma(s)\Gamma(4.5-2s)} \tag{1}$$

where $\rho(r)$ is the charged particle ($e^{\pm}$) track density at a distance, r, from the shower core. The notation follows Ref. [4]:

- $N_e(T_{eff})$ is the total number of particles in the shower at ground level. This varies with the effective shower depth, T_{eff}, in radiation lengths. T_{eff} is subject to substantial shower to shower fluctuations. It depends on the depth of the initial interaction in the atmosphere, on the cosmic ray composition, on the site elevation, on the shower zenith angle and slightly on shower energy [4]. However to the extent that the local pressure is constant then T_{eff} and thus $\underline{N_e(T_{eff})}$ is <u>independent</u> of temperature for any given shower.
- r_m is the Moliere radius in air $\sim$ 2 radiation lengths above the experiment [2]. The Moliere radius depends on the air density. This is modelled assuming the standard atmosphere [5].
- s is the shower age parameter [4,6]. The shower age increases with shower depth and is defined as $s = 1$ at shower maximum. Showers of interest in Auger are in the approximate range $1 \sim s \sim 1.4$. As with $N_e(T_{eff})$, to the extent that the local pressure is constant then T_{eff} and thus $\underline{s}$ is <u>independent</u> of temperature for any given shower.
- The Γ functions, *etc*, normalize $\int_0^\infty \rho(r) 2\pi r dr = N_e(T_{eff})$.

Thus for any given shower, with specified shower energy, angle, T_{eff} and s, $\rho(r)$ will <u>also</u> depend on the local value for $r_{Moliere}$. If $r_{Moliere}$ varies with time then the measured value for $\rho(r)$ will vary with time. Systematic variations in $\rho(r)$ result in *systematic* shifts in the shower energy measurement [1] by the ground array. Systematic shifts for each shower will carry over to systematic shifts in shower averages. Auger must be prepared to correct for these variations. For water Cherenkov tanks, that measure a weighted sum of the $\mu^{\pm}$ and the electro-magnetic components of the shower, the variations in $\rho(r)$ will be less than in this toy study.

The fluctuations in $\rho(r)$ from variations in depth of the initial interaction and from cosmic ray composition are minimal at the atmospheric depth where $\rho(r)$ is a local maximum [4]. However temperature variations in $r_{Moliere}$ shift the maximum value of $\rho(r)$ (for a given shower energy) and thus lead to *systematic* errors in the shower energy measurement as already noted.

It is interesting to note that temperature (and pressure) changes in $r_{Moliere}$ have been both appreciated [8,9,10] and mistakenly ignored [2][13] in the literature. Hillas has emphasized this point [14]. The purpose of the present note is both to remind the collaboration of this issue and to make the point that the temperature [3] variations may be an important *systematic* correction for Auger.

2. The Standard Atmosphere, $r_{Moliere}$ and Analytic Shower Model Predictions

In the model of the standard atmosphere, the atmosphere is characterized by the pressure and temperature at sea level, $P = 1013.25$mb and $T = 288.15$K, plus the adiabatic variation of temperature with altitude, z:

$$T(z) = T(0) - 6.5K \cdot (\frac{z(m)}{1000m})$$

In the standard atmosphere model the pressure at a nominal elevation $z = 1500$m is 846mb. This corresponds to an atmospheric depth of 863gm/cm^2. I use this value as the local pressure for this study. The local temperature is then varied over a likely range between

[1] The Auger experiment will follow Hillas [7] and use the particle density, $\rho(r)$, several hundred meters from the core of an extensive air shower to provide a measure of the shower energy that is relatively insensitive to the depth of the initial cosmic ray interaction in the atmosphere and to the initial composition of the cosmic rays.

[2] When comparing experiments it is essential to make both shower depth corrections (*i.e.* in T_{eff} and s) and corrections in the lateral distribution functions (*e.g.* in $r_{Moliere}$) from local atmospheric differences. The latter varies significantly for the highest energy experiments: Yakutsk $r_{Moliere} \sim 68$m [10], Haverah Park $r_{Moliere} \sim 79$m [11], AGASA $r_{Moliere} \sim 91.6$m [12], and Volcano Ranch $r_{Moliere} \sim 97.5$m [8].

[3] In practice local temperature and pressure will vary and must be monitored and recorded with the Auger data.

the coldest nights in winter to the warmest days in summer.

The Moliere radius, $r_{Moliere}$, is the natural transverse scale set by multiple scattering:

$$r_{Moliere} = \frac{21MeV}{\epsilon_{crit}(MeV)} \cdot X_0$$

where ϵ_{crit} is the critical energy and X_0 is the radiation length of air. This is traditionally evaluated two radiation lengths above the ground array following Greisen [2,8]:

$$r_{Moliere}(m) = 272.5 \cdot \frac{T(K) \cdot (\frac{P(mb) - 73.94 \cdot cos\theta}{P(mb)})^{1./5.25588}}{(P(mb) - 73.94 \cdot cos\theta)}$$

where T and P are the local temperature and pressure, θ is the shower zenith angle, "$73.94 \cdot cos\theta$" corrects to $\sim 2X_0$ above the ground array, and we use the standard atmosphere [5] model to scale the temperature with elevation. While more modern evaluations of $r_{Moliere}$ [15] differ somewhat from Greisen, all that is important for this study is the fractional variation of $r_{Moliere}$ with temperature. We assume that Greisen's evaluation is adequate for this task: for $z = 1500$m, $T = 10°$C and $\theta = 30°$ then $r_{Moliere} = 97.3$m.

Using Eqn. 1, particle densities $\rho(r)$ are simulated [4] at a variety of local temperatures and at four shower energies. Fig. 1a and b show the variations in $\rho(T, r_{dist})/\rho(T = 10°C, r_{dist})$ with temperature at $r_{dist} = 600$m and $r_{dist} = 1000$m respectively. The variations with shower energy are small. The dominant dependance is with temperature where $\sim \pm 10\%$ variation is predicted for $\pm 20°$C temperature variations. While this level of variation might be tolerable on an event by event basis, temperature variations will result in *systematic* shifts in the energies of events summer to winter, and day to night.

In Fig. 2 the particle densities, $\rho(r)$, are plotted versus distance from the shower core for three values of the local temperature: $T = -10°$C, $T = 10°$C, and $T = 30°$C. As in Hillas' prescription for $\rho(500)$ [7], there is a natural distance from the shower core that minimizes the temperature variations in $\rho(r)$. Unfortunately in this instance the distance is ~ 50m from the core rather than 600m or 1000m. At 50m $\rho(r)$ is not well measured by the ground array [16] and $\rho(r)$ will suffer from shower fluctuations [7]. Thus we can not evade the temperature dependance.

As noted above, fluctuations in $\rho(r)$ from variations in depth of the initial interaction and from cosmic ray composition are minimal at the atmospheric depth where $\rho(r)$ is a local maximum [4]. Fig. 3a and b show how $\rho(600)$ and $\rho(1000)$ respectively vary with depth in the atmosphere for a shower of energy 1×10^{20}eV. The three distinct curves in these figures correspond to three values of the local temperature: $T = -10°$C, $T = 10°$C, and $T = 30°$C. Thus temperature variations in $r_{Moliere}$ shift the maximum of $\rho(r)$ for every shower and thus lead to *systematic* errors in the shower energy even in the presence of shower to shower fluctuations.

3. Summary

This study uses simple analytic shower theory to estimate variations in the electro-magnetic component of $\rho(r)$ resulting from temperature changes in the atmosphere over the course of the year. Temperature and pressure dependent variations in $\rho(r)$ are a potential source of *systematic* errors in the ground array measurements that Auger must be prepared to correct in any final analysis. In this toy study, variations in the local temperature of $\pm 20°$C temperature resulted in $\sim \pm 10\%$ variation in the particle densities $\rho(600)$ or $\rho(1000)$. For water Cherenkov tanks, that measure a weighted sum of the $\mu^{\pm}$ and the electro-magnetic components of the shower, the variations in $\rho(r)$ will be less than in this toy study. Nevertheless, the *systematic* variations may be important, and detailed Monte Carlo shower simulations should be run for a range of local temperatures (and probably also pressures) to provide estimates of the corrections for Auger.

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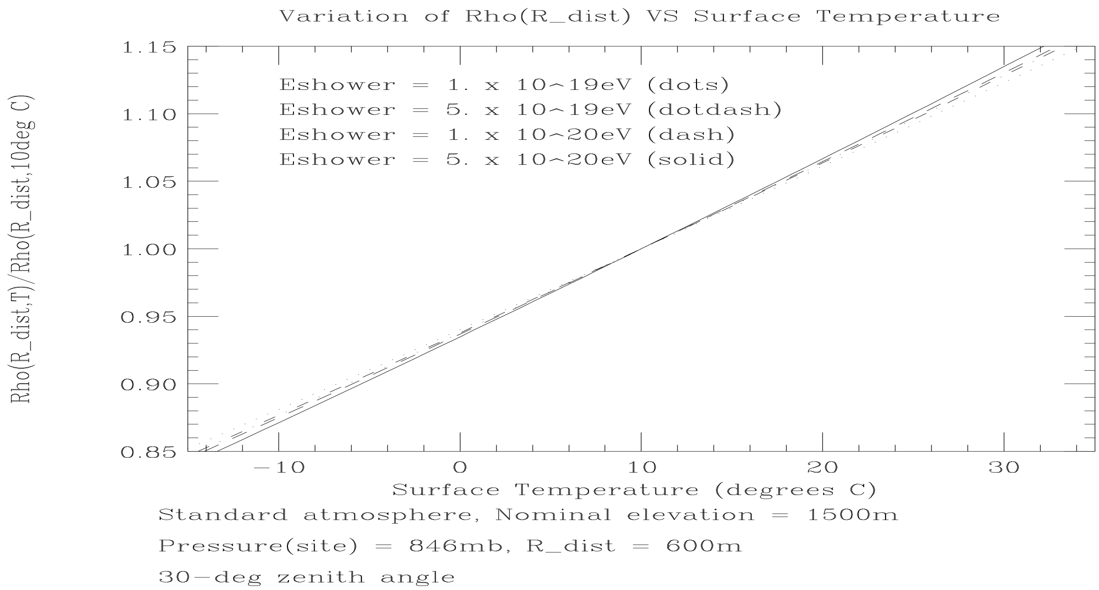

Fig. 1a: Variations in $\rho(T, r_{dist})/\rho(T = 10°C, r_{dist})$ with temperature are shown at $r_{dist} =$ 600m. Particle densities $\rho(r)$ are simulated at four shower energies and at a fixed shower angle $\theta = 30°$.

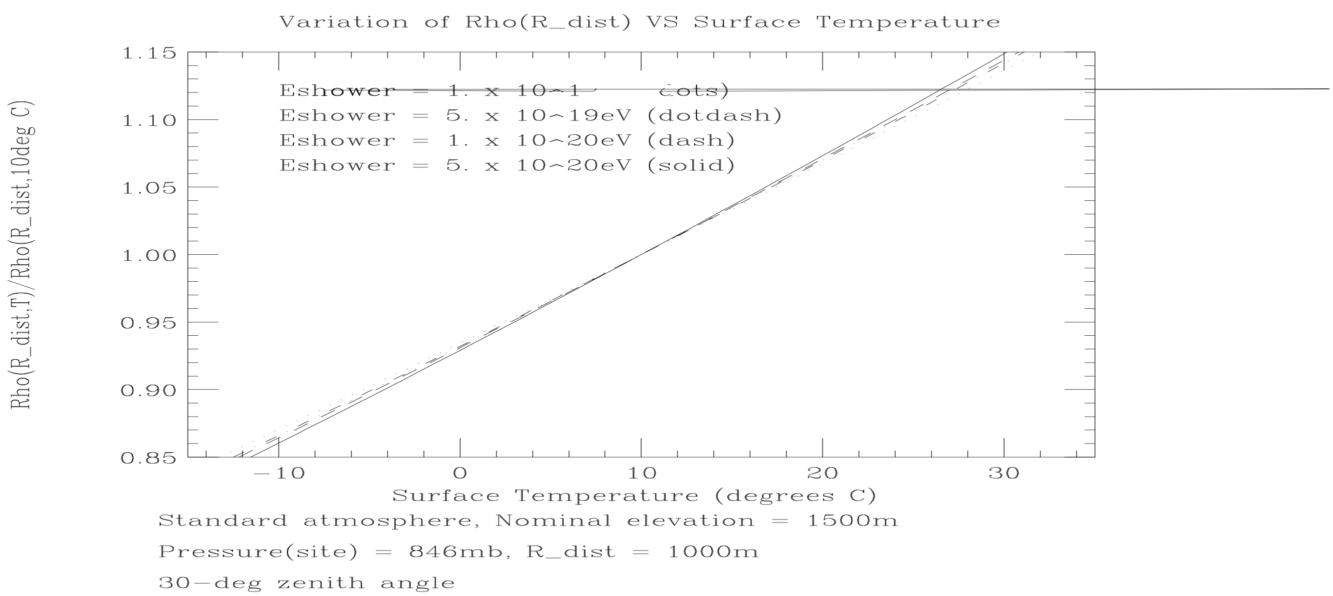

Fig. 1b: Variations in $\rho(T, r_{dist})/\rho(T = 10°C, r_{dist})$ with temperature are shown at $r_{dist} =$ 1000m. Particle densities $\rho(r)$ are simulated at four shower energies and at a fixed shower angle $\theta = 30°$.

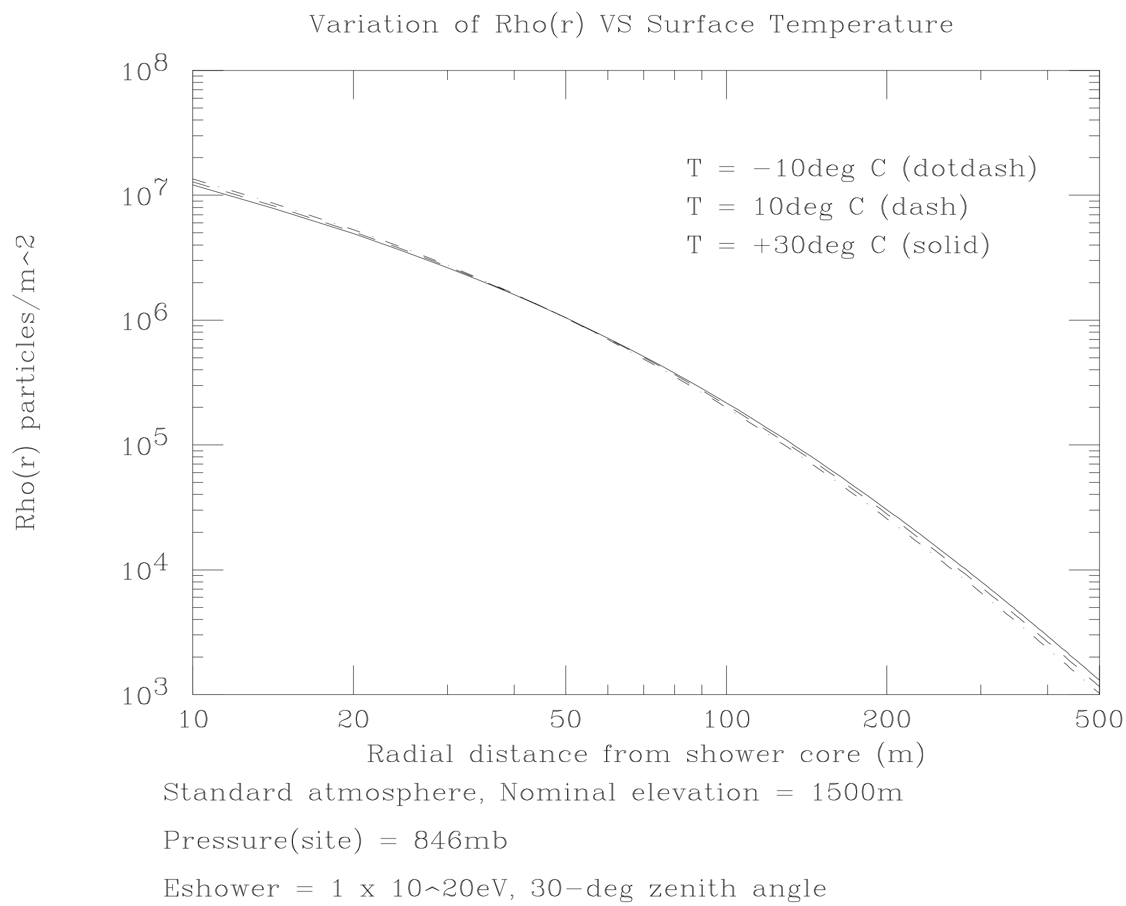

Fig. 2: Particle densities $\rho(r)$ are plotted versus distance from the shower core for three values of the local temperature: $T = -10°\text{C}$, $T = 10°\text{C}$, and $T = 30°\text{C}$. The fixed shower energy and angle are 10^{20}eV and $\theta = 30°$ respectively.

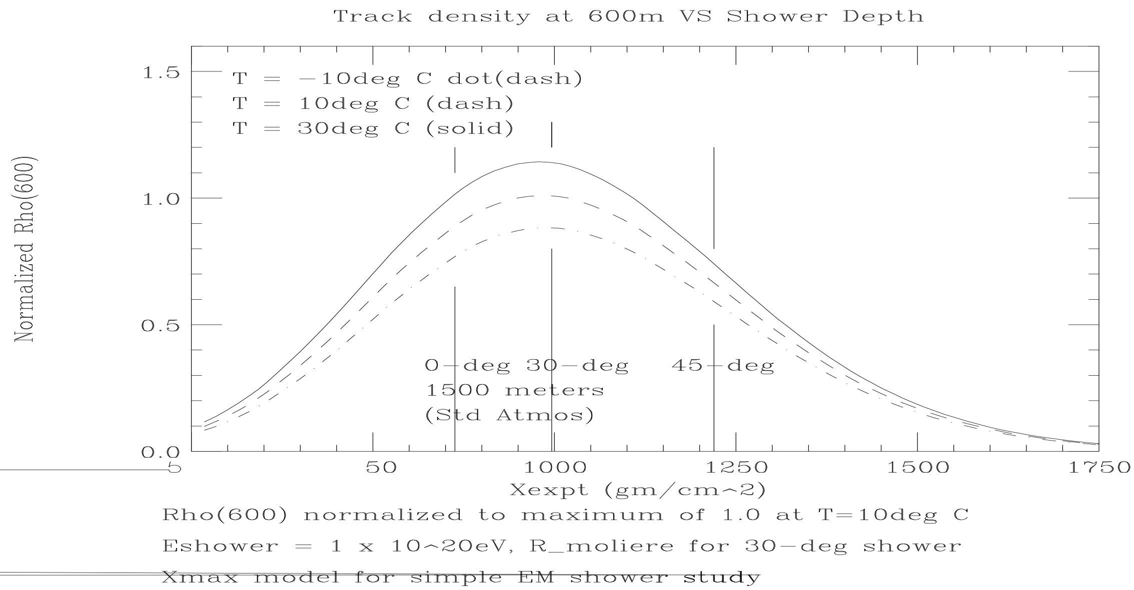

Fig. 3a: Particle densities $\rho(r = 600m)$ are plotted versus shower depth for three values of the local temperature: $T = -10°$C, $T = 10°$C, and $T = 30°$C. The shower energy is 10^{20}eV. The $r_{Moliere}$ length has been evaluated assuming $\theta = 30°$.

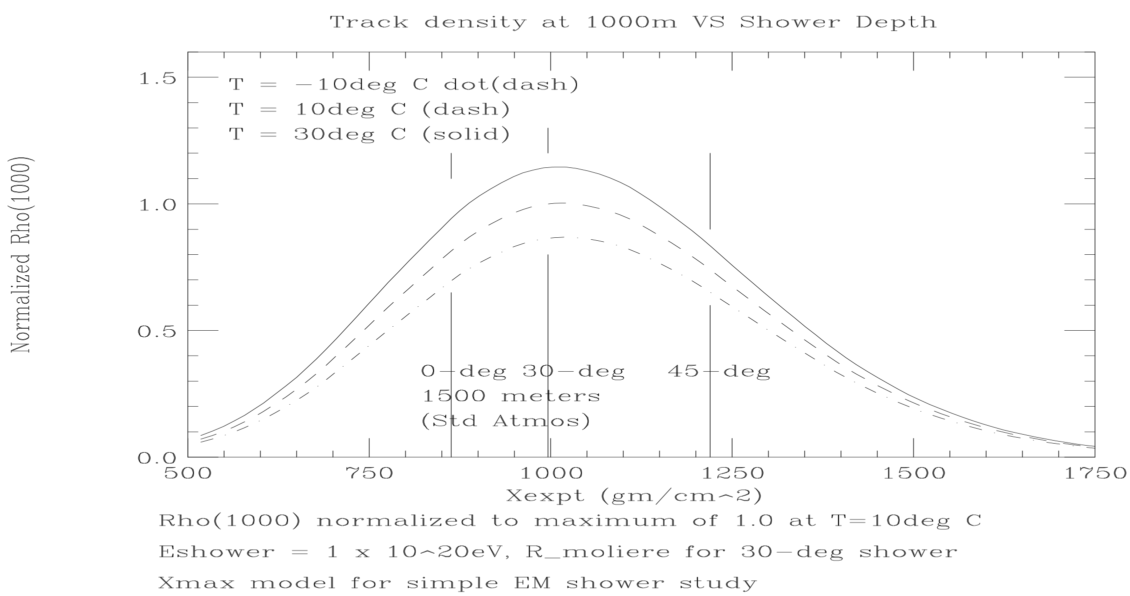

Fig. 3b: Particle densities $\rho(r = 1000m)$ are plotted versus shower depth for three values of the local temperature: $T = -10°$C, $T = 10°$C, and $T = 30°$C. The shower energy is 10^{20}eV. The $r_{Moliere}$ length has been evaluated assuming $\theta = 30°$.