A Parametrization of Cosmic Ray Shower Profiles Based on Shower Width

John A.J. Matthews
johnm@phys.unm.edu
University of New Mexico
Albuquerque, NM 87131

FOR

Bernie Becker, Michael Gold, Doug Hague and Bob Mesler
HiRes-Prototype (2001) Study of Shower Profiles

- **Left plot:** Residuals from comparison of HiRes-Prototype composite shower profile to GIA, Greisen and GH parameterizations. Events have $10^{17} \leq E \leq 10^{18}$ eV.

- **Top plot:** Observed correlation between GH parameters: $T_0 = \frac{X_0}{\lambda}$ and $T_m = \frac{X_{\text{max}}}{\lambda}$.
Conclusion: GH and GIA functions described Corsika showers comparably well.

However GIA was preferable as it required only 3 parameters.

The Monte Carlo study observed the near equality of the width at half-maximum, $f_{whm}$, of proton, iron and photon showers but did not exploit this fact.
A New Approach to Shower Profiles (I)

- How different are the different: GH, GIA and Greisen profiles?
- Are 3-parameters indeed sufficient or are 4-parameters needed?
- Can we profit from similarity of shower $f_{whm}$?
  1. reformulate GH, GIA and Greisen profiles based on: $N_{max}$, $X_{max}$, $f_{whm} \equiv \mathcal{L} + \mathcal{R}$ and shower asymmetry $f \equiv \mathcal{L}/(\mathcal{L} + \mathcal{R})$.
  2. then all profiles depend on two dimensionless ratios: $\epsilon \equiv \Delta/W$ where $\Delta = X - X_{max}$, $W \equiv X_{max} - X_0 = \frac{f_{whm}}{R(f)}$, and $\xi \equiv \frac{W}{\lambda}$, $\sigma$ or $\frac{W}{P_{36.7}}$ where $\xi$ depends only on the asymmetry $f$. 

Auger Collaboration Meeting, Målargue, November 14-20, 2009 – p.4/10
A New Approach to Shower Profiles (II)

- **Left plot:** CONEX simulations suggest that the asymmetry parameter \( f \) may provide some discrimination in primary composition.

- **Note:** as the GIA \( \sigma \) parameter” and the GH \( \left( X_{max} - X_0 \right) / \lambda \) ratio” depend only on the asymmetry \( f \), this echoes the results of: V. Scherini et al (ICRC 2007) and S. Andringa et al (ICRC 2009).

- **Right plot:** But the effect is subtle. The GH shower profiles have \( X_{max} = 725 \) gm/cm\(^2\), \( fwhm = 525 \) gm/cm\(^2\) and three different values of asymmetry: \( f = 0.44, 0.45 \) and \( 0.46 \).
What did we learn? (I)

- **Left plot:** Shower profiles with the same $f_{whm}$ and asymmetry $f$ are almost indistinguishable.

- **Right plot:** The GH and Greisen profiles are systematically below the GIA profile for shower depths well away from shower maximum. Thus shower calorimetric energies evaluated using the GIA function are \( \sim 1\% \) larger than those evaluated using GH or Greisen forms.
What did we learn? (II)

The GH calorimetric shower energy is to a good approximation:

\[ E_{\text{calor}}^{\text{shower}} = \langle dE/dx \rangle \ N_{\text{max}} \ fwhm \ \left( \frac{\xi^{-(\xi+1)} \ e^{\xi} \ \Gamma(\xi + 1)}{R(f)} \right) \]

- **Left plot:** The asymmetry parameter \( f \) dependence, terms in \( (\ ) \), is small
- **Right plot:** Thus \( E_{\text{shower}}^{\text{calor}} \propto N_{\text{max}} \ fwhm \); CONEX simulations are shown for proton, iron and photon showers at \( 10^{18.5} \) eV
What did we learn? (III)

- Shower \((f_{\text{whm}}, f)\) parameters are less correlated than conventional parameters.
- But correlated doesn’t mean that 3 parameters are sufficient ...
Composition and Exotics studies

- Plot of shower asymmetry $f$ VS $X_{max}$ for CONEX simulations of proton, iron, and photon showers near $10^{18.5}$ eV

- As conventional showers have tails mostly to larger values of shower asymmetry $f$, typically associated with showers with larger values of $f_{w,h,m}$, exotic studies are urged to search for showers with smaller values of shower asymmetry: i.e. more asymmetric showers!
Conclusions: Shower Profile based on $f_{\text{whm}}, f$

- For profiles with the same $(f_{\text{whm}}, f)$, the GH and Greisen shower profiles are essentially identical and systemically less than GIA for shower depths away from shower maximum.

- Of the three functions, GH is most convenient as the integral of the GH profile is an analytic function.

- Monte Carlo simulated air showers using CONEX, and parameterized in terms of the new parameters: $(f_{\text{whm}}, f)$, have correlations (between those parameters) greatly reduced over the standard parameterizations e.g. Gaisser-Hillas parameters: $(X_0, \lambda)$.

- This allows shower profile reconstructions to add constraints (if needed) on the mostly uncorrelated parameters $f_{\text{whm}}, f$.

- While not a new result, the CONEX shower simulations suggest that the shower asymmetry parameter, $f$, may have some sensitivity to the incident cosmic ray particle type: e.g. p, C/N/O, Fe or $\gamma$.

- For all the juicy details see: LANL arXiv:0909.4014